Multi-Dimensional Diplomacy

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States often negotiate with each other over more than one issue at the same time. This article presents a model of multidimensional international crisis bargaining. Unlike in the unidimensional case, with two issue dimensions, states can send costless signals about their resolve that have dramatic effects on other states' beliefs and actions. In general, the Target of a threat will be most convinced by a threat over one dimension when the Target is itself thought to be highly resolved over that dimension and not unresolved over the other. States can sometimes convey information with certainty, including that they will go to war unless they get their way on both dimensions. The model also leads to some surprising comparative statics, such as that decreases in the probability that the Target is willing to fight can increase the probability of war.

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"We are not now talking about Laos," President Kennedy told Chairman Khrushchev in their 1961 meeting in Vienna.[1] Laos was a potential front line of conflict between the communist and capitalist worlds, and a place the U.S. had explicitly considered military action that would have lead to a conflict with Soviet Bloc forces. The two sides were still in negotiations over Laos and the eventual outcome there was uncertain and partly dependent upon the credibility of each party.[2] But in Berlin, Kennedy wanted to convince Khurshchev, the United States would not make concessions. Countries often admit that their resolve in one area is not high *relative* to their resolve in some other area.

This form of communication is common in international politics, but its implications have not been appreciated. In negotiations that could result in international conflict, more than one issue is often involved. In the context of negotiating peace-settlements after major wars, so many issues are involved that some are always traded by one side in return for concessions on others. Such negotiations do more than produce agreements. They also give information to the sides about what the other is willing to fight for and what it considers less important. It is the experience of these negotiations, much more than the documents they produce, that form the mutual expectations that are the basis of particular international orders and post-war settlements.

In order to understand the mechanism through which information is communicated, this article considers a bargaining model similar to many found in the literature. No information can be conveyed through costless diplomacy when only one issue is on the table. When two (or more) issues are on the table, this conclusion is reversed. States can sometimes costlessly communicate which issue is more important to them, and can even increase the other side's

[1] *Foreign Relations of the United States*, V. 5, 87.

[2] See Freedman 2000, pp. 293-305.

evaluation of the likelihood that the threatening state would fight for both issues. The dynamics of multi-issue negotiations depend critically on whether the threatened state is thought likely enough to make concessions on one issue, both or neither.

The next section discusses the present concern with multi-dimensional signaling in the context of literature in economics and international relations that addresses similar questions. The following section presents a model of signaling in two dimensions. Implications of the model, along with comparative static results on the probability of conflict, are discussed in a subsequent section.

Multi-Dimensional Diplomacy

Costly threats and actions are often thought to increase the credibility of threats because unresolved states might not be willing to incur the cost.[3] In highly adversarial contexts, threats that do not carry direct costs to the threatener sometimes cannot affect the perceptions of the other side. Such threats are so much "cheap talk" in the colloquial sense of the term.

Recently, scholars have described several mechanisms through which diplomatic exchanges that do not carry direct costs, including those between adversaries behind closed doors, may affect states' perceptions of each others' intentions. Sartori describes how the need to maintain a bargaining reputation lends credibility to threats.[4] Kurizaki shows that states make private threats so as not to arouse public sentiment in threatened states.[5] Guisinger and Smith discuss

[3]See, for instance, Fearon 1995, Schultz 2001, Kydd 1997, 2005, Morrow 1986, 1989, Powell 1988, 1990, 1999.

[4]Sartori 2005.

[5]Kurizaki 2007.

how public and private mechanisms interact.[6] AUTHOR describes communication mechanisms that exist when states have options to prepare for conflict, such as striking first, realigning alliance commitments and draining the resources of the adversary.[7]

This literature draws on the seminal paper by Crawford and Sobel, which characterized cheap talk equilibria in Sender-Receiver games and demonstrated that communication could be more precise when the interests of the players are more closely aligned.[8] More recent work in the economics literature looks at the implications of multiple issue dimensions for signaling in the Crawford-Sobel model.[9] Battaglini shows that in a multidimensional issue space, if there is more than one sender (for example, more than one expert will offer an opinion), full revelation of information is possible though this never occurs in the unidimensional case.[10] The model most related to the current work is Chakraborty and Harbaugh 2007. They show that multidimensional cheap talk can permit information transmission in cases where no information transmission would be possible in a single dimension. The results derived in these papers are

[6]Guisinger and Smith 2002.

[7]AUTHOR. Other scholars who have contributed to our understanding of the mechanisms of information transmission in costless, private, diplomatic exchanges include Jervis 1970; Schelling 1966, 1980; Fearon 1994*a*, 1995, 1997; Nicolson 1963; Kydd 1997; Ramsay 2004; Morrow 1989; Powell 1990; and Der Derian 1987.

[8]Crawford and Sobel 1982.

[9]Interestingly, Aumann and Hart 2003 show that information must sometimes be revealed through discussion. That is, information can sometimes be revealed in equilibrium only after the other side has revealed other information through cheap talk signals of its own. If the signaling stage provides no opportunity for back and forth, these informative equilbria are eliminated. Krishna and Morgan 2004 also show that active costless participation by the uninformed decision-maker in the Crawford-Sobel framework can lead to improved opportunities for communication.

[10]Battaglini 2002.

suggestive, but since they employ a modified Crawford-Sobel framework, they are not directly relevant to international crisis bargaining. The model presented below was developed for this strategic context. In contrast to these models in the economics literature, uncertainty is two sided, both sides take actions that are relevant to each others' utilities, and the adversarial nature of the relationship is the specific sort produced when a side that can make threats must later decide whether or not to engage in costly conflict.[11]

A largely separate literature addresses whether "issue linkage" can facilitate agreement. Some scholars have noted that linkage can sometimes expand the bargaining space such that an agreement that both sides prefer to conflict exists even when no such agreement exists when a single issue is considered in isolation. These scholars emphasize the possibility of gains from trade when the sides differ on which policy dimensions are most important.[12] Morrow 1992 argues, however, that attempting to link issues may signal weakness, in addition to willingness to compromise, impeding states' ability to achieve compromise along these lines.[13]

In the literature on costless diplomacy and issue linkage, states are thought of as explicitly or implicitly making decisions about how to partition the set of political issues into those they will commit to fighting for and those they will not. In the international relations context,

[11]Levy and Razin 2007 establish limits on what can be communicated in a multi-dimensional Crawford-Sobel context. Jackson and Sonnenschein 2007 show that even if agents have uncertainty over each others preferences and incentives to misrepresent it, a Pareto efficient social choice rule can be approximated by an incentive compatible mechanism when the number of similar collective decision problems grows large. In other words, when the decision problem occurs over and over a large number of times, social planners can develop mechanisms to induce agents to reveal their preferences so that the choice rule approximates a Pareto optimal rule.

[12]See, for instance, Morrow 1986, Raiffa 1982, Sebenius 1983, Morgan 1990, Lohmann 1997, Koremenos and Snidal 2001, pp. 786-7, Stein 1980, Lacy and Niou 2004 and Davis 2004.

[13]This issue is addressed further in the Discussion section below.

information is communicated precisely because states sometimes find it in their interest to admit that some issues are not worth fighting for. This gives weight to states' statements that they are willing to fight.

In evaluating each others' intentions, states probably make use of all of the mechanisms of private diplomacy mentioned above. This article presents another mechanism that is available when more than one issue is in contention simultaneously. It is related to the recent literature on multi-dimensional cheap talk, but presents an underlying material strategic context that is significantly different from the sender-receiver games of that literature. The approach taken here is also related to the literature on issue linkage, but rather than emphasizing known gains from trade and expansion of the bargaining space as that literature does, this article focuses on the signaling benefits when more than one issue is on the table.

These questions are important to explore theoretically because, in practice, there are often multiple issues that divide states. There are always multiple areas where the behavior of one state affects the security and interests of another. Sartori discusses U.S. president Johnson's drawing an explicit distinction between the level of U.S. interests in Berlin and Czechoslovakia in 1968. In dropping the U.S. commitment to the former, Johnson emphasized the abiding U.S. commitment to the latter. Sartori explains that if an irresolute Johnson had made a threat over Czechoslovakia, the U.S. could have acquired a reputation for bluffing, making its stance over Berlin less credible.[14] Leaders sometimes think in terms of preserving bargaining reputation, but other dynamics are also at work.

When multiple issues are at stake, states have common interests in communicating with each other so that they can avoid war - whether or not being caught in a bluff will affect their reputation later. Misrepresenting one's own state's interests as highly engaged in an issue

[14]Sartori 2002, p. 122.

when they are not runs the risk of setting countries on a course towards conflicts that may not be necessary. By limiting demands, states make the demands they do make both more credible and more palatable. This allows information to be conveyed through simple, cheap talk mechanisms when signaling is multi-dimensional. Such mechanisms of communication are available even when states are unconcerned about their reputations in future crises.

Most real world negotiations involve more than one issue or involve one only because the parties freely choose to moderate their demands. The Austrian demarche of 1914 contained 10 demands, and the Serbians accepted all but the sixth. When U.S. President Madison considered war in 1812, he demanded both that Britain repeal its commercial Orders in Council and cease impressment of U.S. sailors. When the British acquiesced in one, he still went to war.[15] During the Cuban Missile Crisis, three key issues were negotiated over: the removal of the missiles from Cuba, the Jupiter missiles in Turkey and a public U.S. pledge not to invade Cuba. Before U.S. President George W. Bush declared war on the Taliban in 2001, he demanded the Afghan government both turn over the leaders of Al Qaeda and permanently close terrorist training camps.

In the negotiations at the close of major wars, the points of contention tend to be particularly varied and complex. The negotiations in Paris in 1919 were so complex that the British delegation alone included 217 members. At the Yalta conference in 1945, Russia insisted on shifting Poland to the West; agreed to join the United Nations, but insisted it have a security council with veto power; and agreed to join the war with Japan. In the negotiations to end the Korean War, the territorial demarcation between North and South was one issue, but disagreement over the policy on repatriation of prisoners of war played a large role in preventing a settlement of the conflict for two years. The communist governments were concerned to prevent

[15] See Perkins 1963, Stagg 1976, Hickey 1989, Brown 1964.

a voluntary repatriation in which many captured communist soldiers might choose to remain in South Korea or move to the West.

Crisis bargaining, peace negotiations and other types of diplomacy often involve multiple issues. The model presented below is a crisis bargaining model with two dimensions. It demonstrates that costless or private diplomacy can be effective in circumstances where scholars have previously argued that it could not be.

The Model

The stages of the game are represented in Figure 1, along with two interpretations consistent with the game's underlying structure. In the first interpretation, one state, hereafter the "Deterrer", initially has the opportunity to make costless threats and assurances to the other state (the "Target") by sending signals represented by m. Then the Target decides whether or not to comply with the Deterrer's demands, and finally the Deterrer decides whether or not to go to war ($r \in R \equiv \{0, 1\}$).[16] f_i is player i's utility for going to war, which is not affected by the issues being negotiated over. g_i is player i's utility for peace when it gets everything it wants from the other player. In the second interpretation, we conceive of the Deterrer's final choice of action as the option to accept offers made by the Target in the previous stage or to decline to enter into an agreement. In order to most clearly relate the results derived from the model to previous literature, the presentation of the model below will use the terms of Interpretation 1.

Figure 1 about here.

[16] Broadly similar models can be found in Fearon 1995, 1997; Schultz 2001; Zagare and Kilgour 2000; Signorino and Tarar 2006; Leventoglu and Tarar 2005; and Lewis and Schultz 2003 among others.

If there is only one issue dividing the countries, nothing the Deterrer can say will increase the likelihood the Target decides to back down on the issue. We shall consider the case where two issues are involved. The game tree is shown in Figure 2. The two issues are indexed by $z \in \{1, 2\}$, and the importance of the issue to each side, $i \in \{d, t\}$, is represented by ϵ_i^z, which we assume can take on either a high or a low value. If the countries remain at peace, the players receive the payoffs for peace, g_i, minus their values for whichever of the two issues do not go their way. Whether the sides have high or low values for the issues is private information of each side. We shall use the notation $\underline{\epsilon}_i^z$ for the low value of issue z to player i and $\overline{\epsilon}_i^z$ for the high value. The probability that $\epsilon_i^z = \underline{\epsilon}_i^z$ is $\ell_i^z \in (0, 1)$, and is common knowledge. The ϵ_i^z are independently distributed.

In order to represent the strategic context of negotiating under the threat of violence (or non-agreement under Interpretation 2), we assume $g_d - \overline{\epsilon}_d^z < f_d < g_d - \underline{\epsilon}_d^1 - \underline{\epsilon}_d^2$, so that the Deterrer is willing to go to war if it is a high type with respect to either issue and doesn't get its way with respect to that issue, but unwilling to go to war if it is a low type with respect to both issues and gets its way in neither case. This implies that the Deterrer's most preferred outcome is peace when it gets its way with respect to both issues. We also assume the Target prefers peace when it gets its way: $g_t > f_t$. Thus, the Deterrer is willing to go to war when it considers an issue important, and not otherwise. As we shall see, there are informative equilibria whether high Target types prefer war ($f_t > g_t - \overline{\epsilon}_t^z$) or not.

Figure 2 about here.

The Target's action is $a \in A \equiv \{00, 10, 01, 11\}$, where 10 represents concede on the first issue, but not on the second, and the notation for the three other possible actions follows similar logic. Let $m \in M \equiv A$ represent the Deterrer's signal. It will sometimes be convenient to think of 11 as representing the statement, "both issues are important," 10 the statement, "issue one is important, but issue 2 is not," and so on. There is, however, no inherent meaning to the signals from a game theoretic point of view. Let $\mu(y_d \mid m)$ be the Target's updated beliefs about the Deterrer's type following signal m. Let p_a be the Target's evaluation that the Deterrer will go to war if the Target chooses action a before the crisis begins, and $(p_a \mid m)$ be the posterior probability the Target assigns to war given action a following signal m. Let q_a be the probability the Target chooses action a and $(q_a \mid m)$ be the probability the Target chooses a given signal m.

We shall sometimes refer to a particular player i's type in a unified way as $y_i \in Y_i \equiv \{hh, ll, hl, lh\}$, where hh stands for $\epsilon_i^z = \overline{\epsilon}_i^z \; \forall z$, hl for $\epsilon_i^1 = \overline{\epsilon}_i^1, \epsilon_i^2 = \underline{\epsilon}_i^2$, and so on. $m(y_d)$ is the signal sent by the Deterrer contingent on its type. $r(a, m, y_d)$ is the Deterrer's strategy at the final node, given the Target's action and the Deterrer's own signal and type. $a(m, y_t)$ is the Target's action contingent on the Deterrer's signal and the Target's type. A perfect Bayesian equilibrium of the game is a 4-tuple, $(m^*(y_d), r^*(a, m, y_d), a^*(m, y_t), \mu^*(y_d \mid m))$.[17]

First, we shall examine the logic of informative equilibria through an example. As we shall see, the Deterrer is more likely to get whatever it asks for than it was before it asked (or than it would be in the absence of a communication mechanism). If the Deterrer says that issue 1 is important, but issue 2 is not, for instance, the Target is more likely to respond by backing down on issue 1 and holding firm on issue two than it otherwise would have been. Surprisingly, if the Deterrer says that *both* issues are important, the Target is also more likely to concede on both (1) than it would be if the Deterrer said anything else (2) than it was before the Deterrer said anything at all, and (3) than it would have been in the absence of a communication mechanism.

[17] We ignore the Deterrer's own updated beliefs about the Target's type because the expected utilities associated with the Deterrer's choices at the final stage (where it's beliefs are updated) do not depend on the Target's type.

Suppose $g_i = 50$ and $f_i = 20$. Let $\underline{\epsilon}_i^z = 5$, $\overline{\epsilon}_d^z = 35$ and $\overline{\epsilon}_t^z = 15$. These assumptions imply that high type Deterrer's will fight over an issue and low types will not, so that there is some probability the Deterrer will fight over one or the other of the issues, some probability it will fight unless it gets its way on both and some probability it will not fight even if it gets no concession at all from the Target. The Target will generally prefer a settlement to war, but it also prefers not to back down on either issue. Target types that have a high value for both issues are indifferent between war and a double concession. (As subsequent analysis will demonstrate, none of these assumptions are necessary for the existence of informative equilibria.) To complete the parameterization, suppose there is a 40% chance that $\epsilon_i^z = \underline{\epsilon}_i^z$. In other words, both players believe there is a 40% chance that the other player is a low type with respect to a particular issue (and a 16% chance that the other player is a low type with respect to both issues).

There is then an equilibrium in which: (1) the Deterrer says that issue one (two) is worth fighting for while two (one) is not if and only if that is the case, and the Target concedes the important issue but not the other; (2) the Deterrer says that both issues are worth fighting for when either both are or when neither is. Since the Deterrer only says that one is important and the other not when that is the case, the Target learns from the Deterrer's statements to this effect. When the Deterrer says that both are important, the Target knows that it is useless to make a concession on only 1 issue. Either it makes a concession on both or it makes one on neither.[18] When the Target is a high type on both issues, it makes no concession on either; otherwise, it concedes both. 64% of the time, therefore, the Target concedes both issues when the Deterrer says that both are important.

[18] Off the equilibrium path, if the Deterrer were to admit that both issues are unimportant, the Target would conclude that this is true. Since Deterrer's never prefer to have the Target draw this conclusion, such signals are not sent in equilibrium.

The Deterrer's statements have a dramatic effect on the Target's beliefs about what the Deterrer will do in response to the Target's actions. Before the Deterrer's signal, the Target believes there is a 60% chance the Deterrer will fight over the first issue. After the Deterrer's signal that it will fight for that issue, but not for the other, the Target knows for sure that the Deterrer is willing to fight. Before the sides communicate, the Target believes there is a 36% chance the Deterrer will go to war unless it gets its way on both issues. When the Deterrer says that both issues are important, however, the Target believes there is a 69% chance the Deterrer will fight unless full concessions are made.

Because the Deterrer's statements have such a sizable effect on the Target's beliefs, they also have a sizable effect on the Target's actions. In the absence of a communication mechanism, the Target always makes a concession on one issue and not on the other unless it considers both issues unimportant, in which case it concedes both. If one issue is less important, it concedes the less important issue. If both issues are important, it still concedes one of them, but is indifferent as to which one. Thus, the probability the Target backs down on issue 1, but not on issue two is 42% in the absence of the communication mechanism and 100% following the Deterrer's signal that it will fight for issue one, but not for issue 2.[19] In the absence of a communication mechanism, there is only a 16% chance the Target will make concessions on both issues. In the informative equilibrium, before the Deterrer signals its intentions, there is a 33%[20] chance the Target will back down on both issues; when the Deterrer signals it will fight

[19]The probability the Target chooses 10 is 42% assuming that Targets for whom both issues are important, whose utility is maximized by making a single concession, flip a coin to decide which issue to concede.

[20]In the uninformative equilibrium, only Targets with a low value for both issues concede them both. Thus, the probability of this outcome is $40\% * 40\% = 16\%$. In the informative equilibrium, the probability the Target backs down on both is equal to the probability the Deterrer sends the signal that both are important times the probability the Target backs down contingent upon receiving that signal: $(40\% * 40\% + 60\% * 60\%)64\% = 33\%$.

for both, that probability rises to 64%.

In this equilibrium, Deterrers that are unwilling to fight for either issue claim that they are, as do Deterrers that are willing to fight for both. This seems natural, but why do Deterrers that are willing to fight for only one issue signal this to the Target? Since Deterrers would prefer a concession on both issues even if they are unwilling to fight for one, how can such behavior be optimal? To understand the signaling logic, suppose a Deterrer willing to fight only for issue 1 were to deviate and claim a willingness to fight for both. It would then increase its chances of getting its way on both issues, but *decrease* its chances of getting its way on the issue it really cares about. Its expected value from deviating is its probability of getting its way on both (64%) times its payoff (50) plus its probability of getting neither (36%) times its expected payoff for war (20), which is 39.2. On the other hand, its expected value from its equilibrium strategy of admitting it will only fight over the first issue is the probability it gets its way on the first issue but not the second (100%) times its payoff from that outcome ($50 - 5$), which comes to 45. Thus, by deceiving the Target, the Deterrer would lose its opportunity to convince the Target of its seriousness about the first issue and risk having to go to war to secure what it deems to be truly essential - all for the sake of a chance to attain something it considers relatively unimportant.

We now turn to a more general analysis of the model. Propositions are stated informally in the text and formally above their proofs in the appendix. For the general reader, the informal statements in the text and the surrounding discussion suffice to convey the essential logic of the argument.

The Deterrer's actions can often affect the Target's beliefs and the Target's actions, but how much information is conveyed by particular messages of the Deterrer depends on context. In no context, however, can the Deterrer convey precise information to the Target about its intentions. Proposition 1 tells us that it is impossible for Deterrers that consider neither issue important and some other Deterrer type to each send an honest signal such that the Target knows their types with certainty. At least one of these Deterrer types must pool with other types. A direct consequence is that, in any equilibrium, there is always a signal the Deterrer sends with positive probability that leaves the Target unsure as to the conditions under which the Deterrer would be willing to go to war.

Proposition 1 There is no perfect Bayesian equilibrium in which those unwilling to go to war over either issue send a unique signal and at least one other type (e.g. those willing to go to war over issue 1 but not issue 2) also sends a unique signal.

Proposition 1 implies that in any equilibrium lies occur with positive probability. There is no equilibrium in which all four Deterrer types send different signals so that the Target will always know with certainty who is who after observing the signal. Nevertheless, as the preceding example illustrates, substantial information can often be conveyed.

In order to simplify the following exposition, we shall now restrict attention to the area of the parameter space where Targets that value an issue highly prefer to fight for it rather than concede on that dimension. We shall refer to this as Assumption 1 or A1, which is stated formally below. To gain insight into the negotiating dynamics, we shall study the most informative equilibrium in the game. This is the equilibrium where the highest number of types reveal their true intentions for a given set of parameters. Proposition 2 demonstrates the unique most informative equilibrium of the game under Assumption 1 when ℓ_d^1 and ℓ_d^2 are not both so high that the Deterrer has trouble influencing even the least resolved Targets, and when the Target is itself sufficiently likely to be willing to fight over the two issues.

Assumption 1 (A1) $g_t - \bar{\epsilon}_t^z < f_t \; \forall z$

Proposition 2 Under A1, if the probabilities the Deterrer and Target are not resolved over issue 1 (ℓ_i^1) and issue 2 (ℓ_i^2) are low enough, a unique maximally informative equilibrium (characterized in the appendix and represented in Figure 3) exists.

Figure 3 about here.

The dynamics implied by Proposition 2 are summarized in Figure 3.[21] The figure shows the key role played by the Deterrer's beliefs about how likely the Target is to be willing to fight for each issue (ℓ_t^1 and ℓ_t^2) in determining what the Deterrer will say and what sort of information is conveyed by the statement. ℓ_t^1 is on the horizontal axis and ℓ_t^2 is on the vertical axis. When ℓ_t^2 is above the threshold $T1 = \frac{\epsilon_d^1}{\underline{\epsilon}_d^1+\underline{\epsilon}_d^2}$, the Deterrer can signal with certainty that "issue one is important and issue two is not" and prefers to do so when this is in fact true. When ℓ_t^1 is above the threshold $T2 = \frac{\epsilon_d^2}{\underline{\epsilon}_d^1+\underline{\epsilon}_d^2}$, the Deterrer can signal that "issue two is important and issue one is not". When both ℓ_t^1 and ℓ_t^2 are above the thresholds, the Deterrer can signal with certainty that *either* "issue one is important and issue two is not" or that "issue two is important and issue one is not", but not that both are important.[22] Thus, the credibility of the Deterrer's

[21]In a game theoretic framework, there is no inherent meaning to symbols. If there is an equilibrium where the Target conditions its action on the message, "I will attack unless you concede issue 1," then there is an equivalent equilibrium with the same induced distribution over outcomes in which players condition on the message, "Sally sells sea shells." The meaning of a signal sent in an equilibrium of a game theoretic model is generally clear, however, from which types pool over that message. If the types that are unwilling to go to war send the same message as types that are willing to go to war, and if the former types prefer to be mistaken for the latter while the reverse is not the case, then the message must be something like, "I am a type that is willing to go to war." In the exposition here, we shall use this logic in ascribing meaning to statements. Based on the message and the particular equilibrium dynamics, the Target can draw an inference about the set of possible Deterrer types that would send such a message. We shall interpret the meaning of the message as the Deterrer claiming to be the highest type among the set of types willing to send the particular message. This is for expositional purposes only; the results derived do not depend on ascribing particular meanings to messages.

[22]Of course, if $1 - \frac{\epsilon_d^2}{g_d - f_d} < \frac{\epsilon_d^1}{\underline{\epsilon}_d^1+\underline{\epsilon}_d^2}$ or $1 - \frac{\epsilon_d^1}{g_d - f_d} < \frac{\epsilon_d^2}{\underline{\epsilon}_d^1+\underline{\epsilon}_d^2}$, Region 2 will not exist.

statement that only issue 1 is worth fighting for is facilitated by the increased likelihood that the Target is *less resolved* over the *second issue*. If the Target is thought likely to be unresolved over the first issue but highly resolved over the second, then it conveys less information to claim to be willing to fight only for what the Target is thought likely to be willing to give up and unwilling to fight for what the Target is relatively unlikely to give up. When either ℓ_t^2 is below T1 or ℓ_t^1 is below T2, the Deterrer can signal with certainty that "both are important". Thus, the Deterrer's signal that both are worth fighting for is facilitated by the increased likelihood that both issues are important to the Target.

Proposition 3 Under A1, when the probabilities the Deterrer and Target are not resolved over issue 1 (ℓ_i^1) and issue 2 (ℓ_i^2) are low enough, an equilibrium exists in which, for any Deterrer demand, the Target is more likely to take the action demanded by the Deterrer than the Target was prior to the demand.

Proposition 3 reminds us that in the equilibrium described in Proposition 2 and in Figure 3, whatever statement the Deterrer makes increases the Target's belief that the content of the statement is the case. In other words, if the Deterrer says it will go to war over issue one, but not over issue two, the Target's updated belief following the statement that this is true is strictly higher than it's prior belief before the Deterrer's statement. Thus, when the Deterrer says in Region 3 that it will fight for issue 1, but not for issue 2, although the Target cannot be sure this is the case, the Target believes it more likely to be true than it did prior to the Deterrer's statement. In the example discussed previously, the signaling strategy was that shown in Region 2 of Figure 3. There, when the Deterrer claims to be wililng to fight for 1, but not for 2, the Target is sure that this is the case.

Corollary 1 Under A1, in Regions 1 and 3, an equilibrium exists in which the Target knows for sure that when the Deterrer demands a concession on both issues, the Deterrer will fight unless the Target concedes on both issues.

Surprisingly, even when the Deterrer claims it will fight unless the Target concedes both, the Target concludes this is more likely than it had previously thought. In fact, in Regions 1 and 3, an even stronger statement can be made, formalized in Corollary 1. As long as two issues are one the table, and as long as the Deterrer and Target are believed sufficiently likely to stand firm on both, if the Deterrer says it will fight for both, the Target can be sure the Deterrer is telling the truth. The reason is that Deterrers that are not so resolved on one issue or on either issue realize that they stand a relatively small chance of getting their way even if they claim they will fight for both. If only one is important and they claim that only that one is important, they stand a sufficiently greater chance of getting their way on that issue, making them prefer to signal that one issue is important if one is or that one issue is important even if neither is. For this reason, Targets know for sure that Deterrers that claim they are willing to fight for both really are.

In this model, as in other models in the international relations literature, signaling is effective because of the drawbacks of sending a misleading signal. With costless communication, however, the drawback to a state's misrepresenting itself as highly resolved can be more difficult to see. As in the example, the Deterrer would not want to lie when the lie - even if believed - stands a sufficiently small chance of achieving its end. The Deterrer would not then wish to miss the opportunity to convince the Target about its resolve on the issue it really would go to war over. Alternatively, if it really would go to war over both, it has every incentive to say so. The Deterrer therefore sometimes declines to risk having to go to war in cases where it would be satisfied with peace provided the Target concedes the issue the Deterrer considers most important.

Proposition 4 Under A1, if the probability the Target is not resolved on either issue is high, then at most two signals are sent in equilibrium.

When the probability the Target is not resolved on either issue is high, signaling is less informative. Proposition 2 demonstrates that when ℓ_t^1 and ℓ_t^2 are low (specifically, $\ell_t^1 < 1 - \frac{\epsilon_d^1}{g_d - f_d}$ and $\ell_t^2 < 1 - \frac{\epsilon_d^2}{g_d - f_d}$), an equilibrium can exist in which the four types of Deterrer send three signals with positive probability. This implies that two Deterrer types reveal themselves to the Target - when the Target sees the signal sent by these types, it knows under exactly what conditions the Deterrer will and will not go to war. Proposition 4 shows, however, that when ℓ_t^1 or ℓ_t^2 are high so that it is likely enough that the Target is unwilling to go to war on one issue or the other, then no equilibrium can exist in which such precise information is conveyed to the Target. Then, at most two signals are sent by the four types, which implies that at most one type reveals itself precisely to the Target, implying that signaling is less informative than it can be when the Target is thought likely enough to stand firm on both.

As Proposition 2 makes clear, the conditions on informative signaling are relatively weak. There must merely be sufficient probability that both the Deterrer and Target are in earnest. When both issues are likely to be important to the Target, signaling is particularly effective. This is not to say, however, that informative signaling is impossible outside of Regions 1-3. It is less informative on the whole, but some information can sometimes be conveyed. The parameter values where equally informative equilibria exist overlap, which will sometimes make it even more difficult to predict the signaling dynamics that will most likely occur in those regions. Figure 4 illustrates signaling ranges that are plausible in some cases.

Figure 4 about here.

Discussion

The results above demonstrate signaling benefits of talking about multiple issues at once. The mechanism does not depend on known gains from trade, as in the issue linkage literature, but rather on the information about resolve with respect to one issue that can be inferred from states' willingness to sacrifice a second objective. States are able to communicate which issue is the more important, including - in some cases with certainty - whether or not they are willing to fight for that issue. Surprisingly, by claiming that they will fight for both issues, they are even able to increase a Target's evaluation of the likelihood that this is true.

The mechanism by which information is conveyed is very simple. A particular demand shows a willingness to give up either the possibility of a better deal or the greater chance of achieving a less favorable deal, and in both cases conveys information to the other side. If a state demands multiple concessions, it shows it is willing to give up an increased chance of achieving a settlement in the middle. If a state demands somewhat less, it shows it is willing to give up the chance of getting everything it wants in return for the increased chance of the mid-range settlement. In either case, information is conveyed about what the state considers most important and therefore this information affects the other side's beliefs about what the state is willing to fight for.

It is interesting to note that in the pure strategy equilibria that exist in the area of the parameter space of the model analyzed above, the Deterrer never makes statements such as: "I need a concession on *either* this issue or on the other - we go to war if you're intent on humiliating me in every area," or "why don't we compromise - you decide which issue is resolved in your favor and we'll resolve the other in my favor." This never occurs in a pure strategy equilibrium because of the assumption about types. Since the Deterrer is either willing to fight for an issue or unwilling to fight for it (there are just two Deterrer types for each of the two issues), the

Deterrer is never willing to settle for "either." In a more complicated type space, however, we can imagine that such cases would arise.

As in all costless signaling models, there is also an uninformative equilibrium in which the Deterrer communicates no information to the Target. Here, this relates to Morrow's (1992) argument that offers to link issues may be interpreted as a sign of weakness, which will in turn often prevent states from offering compromises. In the model described here, if the Target believes an offer to split the two issues derives from weakness, and the Deterrer understands this, then no offer to compromise on one but not the other issue will ever be made by the Deterrer. The Target's belief about events that (then) never occur ensures that no information can be communicated through costless statements.

Interestingly, in such equilibria, the Target's belief about the meaning of offers of compromise will never be disconfirmed by experience; no Deterrer will ever offer to compromise. Beliefs of the Deterrer about what signals weakness have a self-fulfilling effect - negotiators who hold such views will never be proven wrong, at least not by direct experience, even though their views are not true in general. This illustrates the subtle sorts of common understandings between negotiators that can have decisive effects on whether they reach agreement. In particular, settlement of disputes will be facilitated if each believes the other does not equate compromise with weakness - at least not in every case.[23] And if the sides each believe this, it will in fact be

[23]Differences of the model presented here with Morrow's (1992) model include: (1) Here, the importance of each issue to each of the sides is private information rather than the Sender's (Deterrer's) value for war and the Receiver's (Target's) value for only one of the issues. For each side, there are 2 sources of private information in the above model rather than 1. (2) Morrow's model incorporates first strike advantages and allows the Target the opportunity to attack. (3) Here, the two issues are always linked and the Target must make a decision on both. In Morrow's model, the Target only makes a decision on one unless the Deterrer offers to link the second issue to the first.

true.[24]

We turn now to one particularly interesting comparative static. One might naturally expect that as the likelihood that the Target is resolved over an issue increases, the probability of war would also increase. This seems particularly likely in a costless signaling model since there would appear to be no selection effect whereby Deterrers refrain from incurring the cost to threaten seemingly resolved Targets. In fact, in a world without communication the probability of conflict declines monotonically in the probability that the Target is unresolved. This is not true of a world with communication.

Figure 5 presents a particular parameterization of the model and illustrates the effect of ℓ_t^1 on the probability of conflict. The top line shows what happens in a world without communication. As it becomes less likely that the Target will not fight for Issue 1, the probability of conflict declines as we would expect. Since in this parameterization the probability the Target will fight for the second issue is fixed at 60%, the overall probability of conflict remains relatively high. In a world with communication, increases in the likelihood that the Target will not fight over the first issue have a more dramatic impact on the likelihood of conflict within a particular signaling region. This can be seen in the decreased slope of the second line in Figure 5. When the probability the Target would not be willing to fight for the first issue increases from 10% to 70%, the probability of conflict decreases from 73% to 43% when the states employ the most

[24] Another limitation of the model is that there are only two sources of uncertainty for each side, namely the other side's evaluation of the importance of each of the two issues before them. In the real world, there are other uncertainties in foreign policy-making. We have also assumed that the level of importance of the two issues to each player are drawn from independent distributions. This will not always be a close representation of the world. As in some arguments for signaling based on reputation, a willingness to concede on one dimension may directly signal a willingness to concede on another dimension. This will of course be most likely when two issues are very similar.

informative communication mechanism available. Without communication, the probability of war decreases only from 78% to 73%.

Figure 5 about here.

In the world with communication, however, the relationship is non-monotonic. This can be seen in Figure 5 from the behavior of the lower line on the right hand side of the Figure. As it becomes more likely that the Target would be willing to make a concession over the first issue, the Deterrer loses the ability to signal as precisely. Rather than sending 3 different signals in equilibrium, depending on its intentions vis-a-vis the two issues, as it does in Regions 1 and 2, the Deterrer is eventually willing to use only 2 signals. In the most informative equilibrium, the Deterrer claims either that only the first issue is important or that both are. When the Target sees the first claim, it does not know for sure whether it is true. When it sees the second, the Target knows the Deterrer would fight for second issue, but not whether it would really fight for the first. This dilution in the precision of the Deterrer's signal causes the probability of conflict to jump discontinuously from below 40% to above 60%, although it remains below the likelihood of conflict in a world without communication (71%). Therefore, quantitative empirical models of the probability of war that include factors that are thought to influence Target commitment on the right hand side of the equation should not assume that these factors have a constant or even monotonic influence on the dependent variable.[25]

[25]The parameterization used to create Figure 5 is: $g_i = 50, f_i = 20, \underline{\epsilon}_d^z = 8, \underline{\epsilon}_t^z = 12.5, \overline{\epsilon}_i^z = 35, \ell_d^z = 30\%, \ell_t^2 = 40\%$. If ℓ_d^z were lower, as ℓ_t^1 increased, there would be a shift from Region 2 to Region 6, as in Figure 4, but since this is not the case, the shift is from the signaling strategy shown in Region 2 to the signaling strategy shown in Region 7.

Empirical Illustrations

The mechanism of inference described here is so simple and intuitive that the reasoning on which it is based is often left implicit. Nevertheless, because the issue environment of international politics is often (and perhaps always) complex, this mechanism is a common way that decision-makers form evaluations of each others' intentions. In intuitive terms, international actors understand that when a state makes a particular demand, it could have made others. Suppose there is a possibility State A will fight rather than make either of two concessions, but that State A is least likely to be willing to make the first of the two concessions. Now suppose that State B demands that State A make the first concession, but not the second. State A will reason that if State B were willing to settle for a concession on the second issue, but not the first, then most likely it would have demanded just that. After all, a demand for the second but not the first is more likely to succeed. Thus, when State B demands the first but not the second, State A knows that State B has given up the opportunity to achieve the more likely concession for the sake of achieving a concession that is less likely to be forthcoming. Making such a choice conveys information to A.

To see how this logic operates in a less abstract setting, consider the following examples. Following the Second World War, the Western powers eventually conceded that Eastern Europe would be in the Soviet sphere of influence. In other areas, however, the West insisted on concessions of its own. Greece was not to be in the Soviet sphere, for instance, and West Berlin was not to be incorporated into East Germany. Insisting on certain concessions in the context of giving up leverage over others conveyed information to the Soviets about the West's resolve vis-a-vis what it did insist on.

Consider, in particular, the Western demand that Berlin not be incorporated into Eastern Germany, which was at the very center of the Cold War. Had the U.S. been willing to compromise, it might have gained concessions in other areas - in Laos and on a nuclear test ban agreement, for instance. By standing firm on the question of Berlin while offering to compromise in other areas, the U.S. demonstrated its willingness to give up the concessions it could have had, had the U.S. itself been willing make concessions over Berlin. The U.S. stance therefore conveyed information to the Soviets, and, as the model suggests, this was particularly true because the U.S. was insisting on something it knew the Soviets were least likely to give up.[26]

We have also seen that demanding concessions on all the issues of the moment can be a meaningful signal that increases an adversary's perception that the demanding state would go to war unless the adversary makes full concessions on all dimensions. To see this aspect of the equilibrium in practice, consider the Middle East War of 1973. On October 6th, Syria and Egypt invaded Israel from the North and South. Following a cease fire, with the Egyptian Third Army surrounded, the Soviet Union made a dramatic threat to intervene in the conflict. In a letter to U.S. President Richard Nixon, Soviet General Secretary Leonid Brezhnev proposed that the Soviet Union and the United States both send combat forces to the region to ensure, "the implementation of the decisions of the Security Council." If the U.S would not act jointly, the Soviet Union would be, "faced with the necessity urgently to consider the question of taking appropriate steps unilaterally."[27] There were thus two issues in this episode of the crisis: whether the U.S. and Soviet Union would act jointly in sending a force and whether, if they did not, the Soviet Union would act unilaterally. This was an immediate deterrence crisis for the United States because all sides understood that the U.S. would prefer the Soviet Union to take

[26]For an analysis of the construction of the Cold War peace settlement, see Trachtenberg 1999. For histories of the conflict over Berlin, see, for instance, Fursenko and Naftali 2006, Freedman 2000 and Smyser 2009.

[27]Brezhnev to Nixon, 24 October 1973. Nixon Presidential Materials Project, Henry Kissinger Office Files, Box 69. For Henry Kissinger's analysis of this moment in the crisis, see Kissinger 2003, p.342.

neither action. The U.S. resolved to demand that the Soviet Union take neither action. The Nixon administration's reply to Brezhnev stated that, "sending Soviet and American military contingents to Egypt is not appropriate... [and] we could in no event accept unilateral action... [which] would produce incalculable consequences which would be in the interest of neither of our countries and which would end all we have striven so hard to achieve."[28]

In demanding that no super power troops enter the conflict zone, either jointly or through unilateral Soviet action, the United States risked war. Had the prospect of joint action been sufficiently palatable vis-a-vis a more conflictual scenario, the U.S. would have accepted the Soviet proposal. In demanding instead that it get its way on both dimensions, the U.S. risked getting its way on neither. From this, the Soviets could conclude that the U.S. was relatively resolved over both dimensions. Therefore, by rejecting the Soviet offer of joint action - even in the midst of Watergate - the U.S. increased Soviet perceptions of U.S. resolve on both questions. Partly as a result, and partly because the Egyptians feared a super power conflict on their soil, the Soviets backed down.[29]

Conclusion

In sender-receiver games in the economics literature, the opportunities for communication are significantly greater if player interaction is over more than a single dimension of conflict. We have seen that the same is true in interstate bargaining, even though the basic structure of the interaction is highly adversarial and even though information is two-sided and both sides

[28]Nixon to Brezhnev, 25 October 1973. Nixon Presidential Materials Project, Henry Kissinger Office Files, box 69.

[29]See Rabinovich 2005 and Kissinger 2003, pp.341-358.

take actions that directly affect each others payoffs. This finding has theoretical and empirical implications. It constitutes another mechanism through which leaders can learn about each others' intentions and suggests that too much is left out of theories that do not account for the effect of diplomacy in shaping perceptions of intentions.

Signaling is informative in multiple dimensions because states lose an opportunity to convince the other side, gain what they consider most important, and avoid conflict when they insist on having their way in every dimension. When the Target and Deterrer are known to be relatively resolved, signals tend to result in a greater shift in the Target's beliefs about the Deterrer's intentions. When the Target is thought sufficiently likely to be willing to fight for an issue, the Deterrer can signal if it is willing to fight for that issue in such a way that the Target knows the truth with certainty. When the Target is thought sufficiently resolved over both issues, it knows the Deterrer is telling the truth when the Deterrer claims it will fight for both.

We also have reason to believe that signaling often is multidimensional. There are many examples of crisis bargaining contexts in which the parties explicitly considered multiple dimensions. Many more cases could be added to those mentioned earlier. Even when only one issue area is addressed by the parties, negotiations may nevertheless have an implicit multidimensional character. Since we have seen that it is sometimes optimal for signaling states to make demands on only one issue, the exclusive focus of a negotiation may be the endogenous result of this strategic process rather an exogenous necessity. In such cases, even though signaling appears unidimensional and costless, it may nevertheless convey very significant information to the sides.

These findings have implications for how we evaluate theories of conflict. Fearon points out strategic selection effects that result in surprising implications when signals carry explicit costs.[30] We have seen here that costless, diplomatic signaling in multiple dimensions also results in surprising comparative statics. We would expect that factors that make it less likely that one state is resolved to fight over an issue would also make conflict less likely. This is true when no communication mechanism exists, but not when states talk to each other and expect each others' signals to convey information. Then, the likelihood of Target resolve has implications for the nature and precision of signaling that create unexpected dynamics. In particular, decreases in the likelihood that a threatened state is resolved to fight over an issue can increase the probability of conflict.

[30] Fearon 1994*a*,*b*.

Appendix

Note on notation: If an argument is left out of a function describing a player's strategy, this will mean that the statement is true for all values of the missing argument, so that $r(11) = 1$ would mean $r(11, m, y_d) = 1\ \forall m, y_d$. If an argument contains several terms in brackets, this will mean the relationship holds for all terms in brackets, so that $a(10, \{hh, ll\}) = 10$ would mean that types hh and ll choose action 10 in response to signal 10. It will sometimes be convenient to refer to player types using the same notation we use for Target actions and Deterrer signals. Therefore, let $n_i : Y_i \to A$ such that $n_i(hh) = 11, n_i(ll) = 00, n_i(hl) = 10,$ and $n_i(lh) = 01$.

Proposition 1 There is no perfect Bayesian equilibrium in which $m^*(ll) \neq m^*(y_d)\ \forall y_d \neq ll$ and $\exists y'_d \neq ll$ such that $m^*(y'_d) \neq m^*(y_d)\ \forall y_d \neq y'_d$.

Proof Suppose not. Then $EU_d(m^*(ll) \mid ll) = g_d - \underline{\epsilon}_d^1 - \underline{\epsilon}_d^2$ which is strictly less than $EU_d(m^*(y'_d) \mid ll)$. To see this, note that $(p_a \mid m^*(y'_d)) = 1\ \forall a \neq n_d(y'_d) \vee 11$. Therefore, $a^*(m^*(y'_d), ll) = n_d(y'_d)$, so that $(q_{n_d(y'_d)} \mid m^*(y'_d)) > 0$. Since type ll Deterrers strictly prefer the outcome following an optimal response to $a(\cdot) = n_d(y')$ to the outcome following its optimal response to $a(\cdot) = 00$, $EU_d(m^*(ll) \mid ll) < EU_d(m^*(y'_d) \mid ll)$, which implies that Deterrer type ll has a profitable deviation, which contradicts our supposition that such an equilibrium exists. ■

Regions In order to state Proposition 2, we first define 3 regions of the parameter space. These correspond to the regions described in Figure 3 in the text. If $\ell_t^1 < 1 - \frac{\underline{\epsilon}_d^1}{g_d - f_d}$ & $\ell_t^2 < 1 - \frac{\underline{\epsilon}_d^2}{g_d - f_d}$,

$$\ell_t^1 > \frac{\underline{\epsilon}_d^2}{\underline{\epsilon}_d^1 + \underline{\epsilon}_d^2} \ \&\ \ell_t^2 > \frac{\underline{\epsilon}_d^1}{\underline{\epsilon}_d^1 + \underline{\epsilon}_d^2} \to \text{The region is R2} \tag{1}$$

$$\ell_t^1 < \frac{\underline{\epsilon}_d^2}{\underline{\epsilon}_d^1 + \underline{\epsilon}_d^2} \vee \ell_t^2 < \frac{\underline{\epsilon}_d^1}{\underline{\epsilon}_d^1 + \underline{\epsilon}_d^2} \to \begin{cases} \text{The region is R1} & \ell_t^2 > \frac{\underline{\epsilon}_d^1}{\underline{\epsilon}_d^2}\ell_t^1 \\ \text{The region is R3} & \ell_t^2 < \frac{\underline{\epsilon}_d^1}{\underline{\epsilon}_d^2}\ell_t^1 \end{cases} \tag{2}$$

Definition 1 Let $h(o) \in H(o)$ index the set of signaling strategies that can be supported by some pure strategy equilibrium given the vector of parameters o. (Note that $H(o)$ is finite.) Let

$n(h(o))$ be the number of signals sent with positive probability under $h(o)$, given parameters o. A pure strategy equilibrium with signaling strategy $h(o)$ is *maximally informative* at o if $n(h(o)) \geq n(h'(o))\ \forall h'(o) \in H(o)$.

Assumption 2 (A2) If $EU_t(a' \mid y_t', m') = EU_t(00 \mid y_t', m')$, the Target chooses $a(y_t', m') = 00$.[31]

Proposition 2 Under A1 and A2, for $\ell_d^2 \leq 1 - \frac{\epsilon_t^2}{g_t - f_t}$ in R1, $\frac{\ell_d^1 \ell_d^2}{(1-\ell_d^1)(1-\ell_d^2)+\ell_d^1\ell_d^2} \leq 1 - \frac{\epsilon_t^1+\epsilon_t^2}{g_t-f_t}$ in R2, and $\ell_d^1 \leq 1 - \frac{\epsilon_t^1}{g_t - f_t}$ in R3, the following strategies and beliefs constitute a maximally informative, pure strategy equilibrium of the game in regions R1-R3. Further, this equilibrium is unique in the sense that no other maximally informative pure strategy equilibrium induces a different distribution over outcomes.

Deterrer's Strategy: The signaling component of the Deterrer's strategy is as follows. Let $m^1 \neq m^2 \neq m^3 \neq m^4 \in M$ be arbitrary messages. In R2, play $m^3(ll, hh), m^2(lh), m^1(hl)$. In R1, play $m^2(ll, lh), m^1(hl), m^3(hh)$. In R3, play $m^1(ll, hl), m^2(lh), m^3(hh)$. Let the 2nd component of the Deterrer's strategy be $r(11) = 0$, $r(00) = 1\ \forall y_d \neq ll$, $r(00, ll) = 0$, $r(01, \{ll, lh\}) = 0$, $r(01, \{hh, hl\}) = 1$, $r(10, \{ll, hl\}) = 0$, $r(10, \{hh, lh\}) = 1$.

Target's Strategy: Let the Target's strategy be $a(m^3, \{lh, hl, hh\}) = 00$, $a(m^3, ll) = 11$, $a(m^1, \{hh, hl\}) = 00$, $a(m^1, \{ll, lh\}) = 10$, $a(m^2, \{hh, lh\}) = 00$, $a(m^2, \{ll, hl\}) = 01$, $a(m^4) = 00$.

Target's Beliefs: We shall specify the Target's posterior beliefs given the Deterrer's signal in terms of the likelihood that the Deterrer will go to war, but note that there is an immediate translation to updated beliefs defined over Deterrer types, $\mu(y_d \mid m)$. $(p_{11} \mid m) = 0\ \forall m$ and $(p_a \mid m^4) = 0\ \forall a$. In R1, $(p_{10} \mid m^3) = (p_{01} \mid m^3) = (p_{01} \mid m^1) = (p_{00} \mid m^1) = (p_{00} \mid m^3) = 1$,

[31] If the Target is indifferent between making concessions and not making them because it knows war will result either way, we shall assume it does not make concessions. This assumption, made in Proposition 2, has little substantive importance but greatly simplifies the cases to consider in the proof.

$(p_{10} \mid m^2) = (p_{00} \mid m^2) = 1 - \ell_d^2$, $(p_{01} \mid m^2) = (p_{10} \mid m^1) = 0$. In R2, $(p_{00} \mid m^1) = (p_{00} \mid m^2) = (p_{01} \mid m^1) = (p_{10} \mid m^2) = 1$, $(p_{01} \mid m^3) = (p_{10} \mid m^3) = (p_{00} \mid m^3) = \frac{(1-\ell_d^1)(1-\ell_d^2)}{\ell_d^1 \ell_d^2 + (1-\ell_d^1)(1-\ell_d^2)}$, $(p_{10} \mid m^1) = (p_{01} \mid m^2) = 0$. In R3, $(p_{10} \mid m^3) = (p_{01} \mid m^3) = (p_{10} \mid m^2) = (p_{00} \mid m^2) = (p_{00} \mid m^{11}) = 1$, $(p_{01} \mid m^1) = (p_{00} \mid m^1) = 1 - \ell_d^1$, $(p_{10} \mid m^1) = (p_{01} \mid m^2) = 0$.

Proof First, note that by Proposition 1, the four types of Deterrer never send four signals in equilibrium, so that if 3 signals are sent for particular parameter values in a particular equilibrium, that equilibrium is at least as informative as any other over that parameter range. For four types, there are six ways to partition types into 3 groups that constitute possible signaling strategies. Of these, three are impossible by Proposition 2 because both Deterrer type ll and another Deterrer type send a signal that no other types send. That leaves three other possible equilibrium signaling strategies, namely those assigned to R1, R2 and R3.

For the Deterrer's signaling strategy to be optimal, 12 conditions must hold, which we represent as follows:

$$EU_d(m^*(y_d) \mid y_d) \geq EU_d(m' \mid y_d) \; \forall y_d, m' \neq m^* \tag{3}$$

Note that in R1-R3, given the Target's strategy, $EU_d(m^*(y_d) \mid y_d) \geq EU_d(m^4 \mid y_d) \; \forall y_d$, which means that an equlibrium must satisfy only the remaining 8 conditions represented in (3).

We have the following expressions for the Deterrer's utility.

$$EU_d(m^3 \mid hh) = (q_{11} \mid m^3)g_d + (1 - (q_{11} \mid m^3))f_d$$

$$EU_d(m^1 \mid hh) = (q_{11} \mid m^1)g_d + (1 - (q_{11} \mid m^1))f_d$$

$$EU_d(m^2 \mid hh) = (q_{11} \mid m^2)g_d + (1 - (q_{11} \mid m^2))f_d$$

$$EU_d(m^3 \mid hl) = (q_{11} \mid m^3)g_d + (q_{10} \mid m^3)(g_d - \underline{\epsilon}_d^2) + (1 - (q_{11} \mid m^3) - (q_{10} \mid m^3))f_d$$

$$EU_d(m^1 \mid hl) = (q_{11} \mid m^1)g_d + (q_{10} \mid m^1)(g_d - \underline{\epsilon}_d^2) + (1 - (q_{11} \mid m^1) - (q_{10} \mid m^1))f_d$$

$$EU_d(m^2 \mid hl) = (q_{11} \mid m^2)g_d + (q_{10} \mid m^2)(g_d - \epsilon_d^2) + (1 - (q_{11} \mid m^2) - (q_{10} \mid m^2))f_d$$

$$EU_d(m^3 \mid lh) = (q_{11} \mid m^3)g_d + (q_{01} \mid m^3)(g_d - \epsilon_d^1) + (1 - (q_{11} \mid m^3) - (q_{01} \mid m^3))f_d$$

$$EU_d(m^1 \mid lh) = (q_{11} \mid m^1)g_d + (q_{01} \mid m^1)(g_d - \epsilon_d^1) + (1 - (q_{11} \mid m^1) - (q_{01} \mid m^1))f_d$$

$$EU_d(m^2 \mid lh) = (q_{11} \mid m^2)g_d + (q_{01} \mid m^2)(g_d - \epsilon_d^1) + (1 - (q_{11} \mid m^2) - (q_{01} \mid m^2))f_d$$

$$\begin{aligned} EU_d(m^3 \mid ll) = (q_{11} \mid m^3)g_d + (q_{01} \mid m^3)(g_d - \epsilon_d^1) + (q_{10} \mid m^3)(g_d - \epsilon_d^2) \\ +(1 - (q_{11} \mid m^3) - (q_{01} \mid m^3) - (q_{10} \mid m^3))(g_d - \epsilon_d^1 - \epsilon_d^2) \end{aligned}$$

$$\begin{aligned} EU_d(m^1 \mid ll) = (q_{11} \mid m^1)g_d + (q_{01} \mid m^1)(g_d - \epsilon_d^1) + (q_{10} \mid m^1)(g_d - \epsilon_d^2) \\ +(1 - (q_{11} \mid m^1) - (q_{01} \mid m^1) - (q_{10} \mid m^1))(g_d - \epsilon_d^1 - \epsilon_d^2) \end{aligned}$$

$$\begin{aligned} EU_d(m^2 \mid ll) = (q_{11} \mid m^2)g_d + (q_{01} \mid m^2)(g_d - \epsilon_d^1) + (q_{10} \mid m^2)(g_d - \epsilon_d^2) \\ +(1 - (q_{11} \mid m^2) - (q_{01} \mid m^2) - (q_{10} \mid m^2))(g_d - \epsilon_d^1 - \epsilon_d^2) \end{aligned}$$

We shall use the notation $I_{y_t}^a(m) = \{x\}$ to mean that $I_{y_t}^a(m) = 1$ if condition x is satisfied and 0 otherwise. These shall be used as indicator variables that equal 1 when type y_t takes action a in equilibrium.

In the R1 case, the Target plays 11 iff the Target's type is ll and the Deterrer sends m^3. Therefore, Deterrer type hh's expected utility from m^3 is strictly higher than it's utility from the other signals, which means 6 conditions remain from equation (3) that the Deterrer's strategy must be shown to satisfy. Note that $(q_{11} \mid m^1) = (q_{11} \mid m^2) = 0$ given the Deterrer's signaling strategy because the Target strictly prefers $a(y_t, m^1) = 10$ to $a(y_t, m^1) = 11$ and $a(y_t, m^2) = 01$ to $a(y_t, m^2) = 11$ $\forall y_t$ since $(p_{10} \mid m^1) = (p_{01} \mid m^2) = 0$. Further, using A2, $(q_{10} \mid m^3) = (q_{01} \mid m^3) = (q_{10} \mid m^2) = (q_{01} \mid m^1) = 0$. Taking these requirements into account, substituting into (3) and simplifying yields the following 4 equations.

$$\frac{q_{11} \mid m^3}{q_{10} \mid m^1} = \frac{\ell_t^1\ell_t^2 I_{ll}^{11}(m^3) + (1-\ell_t^1)(1-\ell_t^2)I_{hh}^{11}(m^3) + \ell_t^1(1-\ell_t^2)I_{lh}^{11}(m^3) + (1-\ell_t^1)\ell_t^2 I_{hl}^{11}(m^3)}{\ell_t^1\ell_t^2 I_{ll}^{10}(m^1) + (1-\ell_t^1)(1-\ell_t^2)I_{hh}^{10}(m^1) + \ell_t^1(1-\ell_t^2)I_{lh}^{10}(m^1) + (1-\ell_t^1)\ell_t^2 I_{hl}^{10}(m^1)} \leq 1 - \frac{\epsilon_d^2}{g_d - f_d} \tag{4}$$

$$\frac{q_{11} \mid m^3}{q_{01} \mid m^2} = \frac{\ell_t^1 \ell_t^2 I_{ll}^{11}(m^3) + (1-\ell_t^1)(1-\ell_t^2) I_{hh}^{11}(m^3) + \ell_t^1(1-\ell_t^2) I_{lh}^{11}(m^3) + (1-\ell_t^1)\ell_t^2 I_{hl}^{11}(m^3)}{\ell_t^1 \ell_t^2 I_{ll}^{01}(m^2) + (1-\ell_t^1)(1-\ell_t^2) I_{hh}^{01}(m^2) + \ell_t^1(1-\ell_t^2) I_{lh}^{01}(m^2) + (1-\ell_t^1)\ell_t^2 I_{hl}^{01}(m^2)} \leq 1 - \frac{\underline{\epsilon}_d^1}{g_d - f_d} \tag{5}$$

$$\frac{q_{11} \mid m^3}{q_{01} \mid m^2} = \frac{\ell_t^1 \ell_t^2 I_{ll}^{11}(m^3) + (1-\ell_t^1)(1-\ell_t^2) I_{hh}^{11}(m^3) + \ell_t^1(1-\ell_t^2) I_{lh}^{11}(m^3) + (1-\ell_t^1)\ell_t^2 I_{hl}^{11}(m^3)}{\ell_t^1 \ell_t^2 I_{ll}^{01}(m^2) + (1-\ell_t^1)(1-\ell_t^2) I_{hh}^{01}(m^2) + \ell_t^1(1-\ell_t^2) I_{lh}^{01}(m^2) + (1-\ell_t^1)\ell_t^2 I_{hl}^{01}(m^2)} \leq \frac{\underline{\epsilon}_d^2}{\underline{\epsilon}_d^1 + \underline{\epsilon}_d^2} \tag{6}$$

$$\frac{q_{01} \mid m^2}{q_{10} \mid m^1} = \frac{\ell_t^1 \ell_t^2 I_{ll}^{01}(m^2) + (1-\ell_t^1)(1-\ell_t^2) I_{hh}^{01}(m^2) + \ell_t^1(1-\ell_t^2) I_{lh}^{01}(m^2) + (1-\ell_t^1)\ell_t^2 I_{hl}^{01}(m^2)}{\ell_t^1 \ell_t^2 I_{ll}^{10}(m^1) + (1-\ell_t^1)(1-\ell_t^2) I_{hh}^{10}(m^1) + \ell_t^1(1-\ell_t^2) I_{lh}^{10}(m^1) + (1-\ell_t^1)\ell_t^2 I_{hl}^{10}(m^1)} \geq \frac{\underline{\epsilon}_d^1}{\underline{\epsilon}_d^2} \tag{7}$$

Under the messaging strategy assigned to R1 and Assumption 1, and assuming the Target uses Bayes' rule to update its beliefs about the Deterrer's types, we know that, $I_{ll}^{11}(m^3) = \{g_t - \underline{\epsilon}_t^1 - \underline{\epsilon}_t^2 \geq f_t\} = 1$, $I_{hh}^{11}(m^3) = \{g_t - \bar{\epsilon}_t^1 - \bar{\epsilon}_t^2 \geq f_t\} = 0$, $I_{lh}^{11}(m^3) = 0$, $I_{hl}^{11}(m^3) = 0$, $I_{ll}^{10}(m^1) = 1$, $I_{hh}^{10}(m^1) = \{g_t - \bar{\epsilon}_t^1 \geq f_t\} = 0$, $I_{lh}^{10}(m^1) = 1$, and $I_{hl}^{10}(m^1) = \{g_t - \bar{\epsilon}_t^1 \geq f_t\} = 0$. Further, for $\ell_d^2 \leq 1 - \frac{\underline{\epsilon}_t^2}{g_t - f_t}$, $I_{ll}^{01}(m^2) = \{g_t - \underline{\epsilon}_t^2 \geq (1-\ell_d^2) f_t + \ell_d^2 g_t\} = 1$, $I_{hh}^{01}(m^2) = 0$, $I_{lh}^{01}(m^2) = 0$, $I_{hl}^{01}(m^2) = \{g_t - \underline{\epsilon}_t^2 \geq (1-\ell_d^2) f_t + \ell_d^2 g_t\} = 1$. Substituting into the above conditions yields the closure of R1. Thus, the closure of R1 is a necessary condition for this signaling strategy to be optimal for the Deterrer. Further, we have now derived the Target's beliefs and optimal response from the Deterrer's strategy and Bayes' rule (and note that these correspond to the strategies and beliefs assigned to R1 in the proposition). This implies that, given the conditions specified in the proposition, R1 is sufficient for the existence of the R1 portion of the equilibrium.

Similarly, in R2, substituting into (3), simplifying and using A2, this again implies 4 conditions:

$$\frac{q_{11} \mid m^3}{q_{10} \mid m^1} = \frac{\ell_t^1 \ell_t^2 I_{ll}^{11}(m^3) + (1-\ell_t^1)(1-\ell_t^2) I_{hh}^{11}(m^3) + \ell_t^1(1-\ell_t^2) I_{lh}^{11}(m^3) + (1-\ell_t^1)\ell_t^2 I_{hl}^{11}(m^3)}{\ell_t^1 \ell_t^2 I_{ll}^{10}(m^1) + (1-\ell_t^1)(1-\ell_t^2) I_{hh}^{10}(m^1) + \ell_t^1(1-\ell_t^2) I_{lh}^{10}(m^1) + (1-\ell_t^1)\ell_t^2 I_{hl}^{10}(m^1)} \leq 1 - \frac{\underline{\epsilon}_d^2}{g_d - f_d} \tag{8}$$

$$\frac{q_{11} \mid m^3}{q_{01} \mid m^2} = \frac{\ell_t^1 \ell_t^2 I_{ll}^{11}(m^3) + (1-\ell_t^1)(1-\ell_t^2) I_{hh}^{11}(m^3) + \ell_t^1(1-\ell_t^2) I_{lh}^{11}(m^3) + (1-\ell_t^1)\ell_t^2 I_{hl}^{11}(m^3)}{\ell_t^1 \ell_t^2 I_{ll}^{01}(m^2) + (1-\ell_t^1)(1-\ell_t^2) I_{hh}^{01}(m^2) + \ell_t^1(1-\ell_t^2) I_{lh}^{01}(m^2) + (1-\ell_t^1)\ell_t^2 I_{hl}^{01}(m^2)} \leq 1 - \frac{\underline{\epsilon}_d^1}{g_d - f_d} \tag{9}$$

$$\frac{q_{11} \mid m^3}{q_{10} \mid m^1} = \frac{\ell_t^1 \ell_t^2 I_{ll}^{11}(m^3) + (1-\ell_t^1)(1-\ell_t^2) I_{hh}^{11}(m^3) + \ell_t^1(1-\ell_t^2) I_{lh}^{11}(m^3) + (1-\ell_t^1)\ell_t^2 I_{hl}^{11}(m^3)}{\ell_t^1 \ell_t^2 I_{ll}^{10}(m^1) + (1-\ell_t^1)(1-\ell_t^2) I_{hh}^{10}(m^1) + \ell_t^1(1-\ell_t^2) I_{lh}^{10}(m^1) + (1-\ell_t^1)\ell_t^2 I_{hl}^{10}(m^1)} \geq \frac{\underline{\epsilon}_d^1}{\underline{\epsilon}_d^1 + \underline{\epsilon}_d^2} \tag{10}$$

$$\frac{q_{11} \mid m^3}{q_{01} \mid m^2} = \frac{\ell_t^1 \ell_t^2 I_{ll}^{11}(m^3) + (1-\ell_t^1)(1-\ell_t^2) I_{hh}^{11}(m^3) + \ell_t^1(1-\ell_t^2) I_{lh}^{11}(m^3) + (1-\ell_t^1)\ell_t^2 I_{hl}^{11}(m^3)}{\ell_t^1 \ell_t^2 I_{ll}^{01}(m^2) + (1-\ell_t^1)(1-\ell_t^2) I_{hh}^{01}(m^2) + \ell_t^1(1-\ell_t^2) I_{lh}^{01}(m^2) + (1-\ell_t^1)\ell_t^2 I_{hl}^{01}(m^2)} \geq \frac{\underline{\epsilon}_d^2}{\underline{\epsilon}_d^1 + \underline{\epsilon}_d^2} \tag{11}$$

Using Bayes' rule to update the Target's beliefs, optimality implies the following for the Target's strategy in this region. For $\frac{\ell_d^1 \ell_d^2}{(1-\ell_d^1)(1-\ell_d^2)+\ell_d^1\ell_d^2} \leq 1 - \frac{\underline{\epsilon}_t^1 + \underline{\epsilon}_t^2}{g_t - f_t}$, $I_{ll}^{11}(m^3) = \{g_t - \underline{\epsilon}_t^1 - \underline{\epsilon}_t^2 \geq \frac{(1-\ell_d^1)(1-\ell_d^2)}{(1-\ell_d^1)(1-\ell_d^2)+\ell_d^1\ell_d^2} f_t + (1 - \frac{(1-\ell_d^1)(1-\ell_d^2)}{(1-\ell_d^1)(1-\ell_d^2)+\ell_d^1\ell_d^2}) g_t\} = 1$, $I_{y_t}^{11}(m^3) = 0 \ \forall y_t \neq 00$. Further, $I_{ll}^{10}(m^1) = 1$, $I_{hh}^{10}(m^1) = \{g_t - \bar{\epsilon}_t^1 \geq f_t\} = 0$, $I_{lh}^{10}(m^1) = 1$, $I_{hl}^{10}(m^1) = \{g_t - \bar{\epsilon}_t^1 \geq f_t\} = 0$, $I_{ll}^{01}(m^2) = 1$, $I_{hh}^{01}(m^2) = \{g_t - \bar{\epsilon}_t^2 \geq f_t\} = 0$, $I_{lh}^{01}(m^2) = \{g_t - \bar{\epsilon}_t^2 \geq f_t\} = 0$, $I_{hl}^{01}(m^2) = 1$. Substituting into the above conditions yields the closure of R2.

In R3, the four conditions are:

$$\frac{q_{11} \mid m^3}{q_{10} \mid m^1} = \frac{\ell_t^1 \ell_t^2 I_{ll}^{11}(m^3) + (1-\ell_t^1)(1-\ell_t^2) I_{hh}^{11}(m^3) + \ell_t^1(1-\ell_t^2) I_{lh}^{11}(m^3) + (1-\ell_t^1)\ell_t^2 I_{hl}^{11}(m^3)}{\ell_t^1 \ell_t^2 I_{ll}^{10}(m^1) + (1-\ell_t^1)(1-\ell_t^2) I_{hh}^{10}(m^1) + \ell_t^1(1-\ell_t^2) I_{lh}^{10}(m^1) + (1-\ell_t^1)\ell_t^2 I_{hl}^{10}(m^1)} \leq 1 - \frac{\underline{\epsilon}_d^2}{g_d - f_d} \tag{12}$$

$$\frac{q_{11} \mid m^3}{q_{01} \mid m^2} = \frac{\ell_t^1 \ell_t^2 I_{ll}^{11}(m^3) + (1-\ell_t^1)(1-\ell_t^2) I_{hh}^{11}(m^3) + \ell_t^1(1-\ell_t^2) I_{lh}^{11}(m^3) + (1-\ell_t^1)\ell_t^2 I_{hl}^{11}(m^3)}{\ell_t^1 \ell_t^2 I_{ll}^{01}(m^2) + (1-\ell_t^1)(1-\ell_t^2) I_{hh}^{01}(m^2) + \ell_t^1(1-\ell_t^2) I_{lh}^{01}(m^2) + (1-\ell_t^1)\ell_t^2 I_{hl}^{01}(m^2)} \leq 1 - \frac{\underline{\epsilon}_d^1}{g_d - f_d} \tag{13}$$

$$\frac{q_{11} \mid m^3}{q_{10} \mid m^1} = \frac{\ell_t^1 \ell_t^2 I_{ll}^{11}(m^3) + (1-\ell_t^1)(1-\ell_t^2) I_{hh}^{11}(m^3) + \ell_t^1(1-\ell_t^2) I_{lh}^{11}(m^3) + (1-\ell_t^1)\ell_t^2 I_{hl}^{11}(m^3)}{\ell_t^1 \ell_t^2 I_{ll}^{10}(m^1) + (1-\ell_t^1)(1-\ell_t^2) I_{hh}^{10}(m^1) + \ell_t^1(1-\ell_t^2) I_{lh}^{10}(m^1) + (1-\ell_t^1)\ell_t^2 I_{hl}^{10}(m^1)} \leq \frac{\underline{\epsilon}_d^1}{\underline{\epsilon}_d^1 + \underline{\epsilon}_d^2} \tag{14}$$

$$\frac{q_{10} \mid m^1}{q_{01} \mid m^2} = \frac{\ell_t^1 \ell_t^2 I_{ll}^{10}(m^1) + (1-\ell_t^1)(1-\ell_t^2) I_{hh}^{10}(m^1) + \ell_t^1(1-\ell_t^2) I_{lh}^{10}(m^1) + (1-\ell_t^1)\ell_t^2 I_{hl}^{10}(m^1)}{\ell_t^1 \ell_t^2 I_{ll}^{01}(m^2) + (1-\ell_t^1)(1-\ell_t^2) I_{hh}^{01}(m^2) + \ell_t^1(1-\ell_t^2) I_{lh}^{01}(m^2) + (1-\ell_t^1)\ell_t^2 I_{hl}^{01}(m^2)} \geq \frac{\underline{\epsilon}_d^2}{\underline{\epsilon}_d^1} \tag{15}$$

Using Bayes' rule to update the Target's beliefs, optimality implies the following for the Target's strategy in this region. $I_{ll}^{11}(m^3) = \{g_t - \underline{\epsilon}_t^1 - \underline{\epsilon}_t^2 \geq f_t\} = 1$, $I_{hh}^{11}(m^3) = \{g_t - \bar{\epsilon}_t^1 - \bar{\epsilon}_t^2 \geq f_t\} = 0$, $I_{lh}^{11}(m^3) = 0$, $I_{hl}^{11}(m^3) = 0$, $I_{ll}^{01}(m^2) = 1$, $I_{hh}^{01}(m^2) = \{g_t - \bar{\epsilon}_t^2 \geq f_t\} = 0$, $I_{lh}^{01}(m^2) = \{g_t - \bar{\epsilon}_t^2 \geq f_t\} = 0$, $I_{hl}^{01}(m^2) = 1$. For $\ell_d^1 \leq 1 - \frac{\underline{\epsilon}_t^1}{g_t - f_t}$, $I_{ll}^{10}(m^1) = \{g_t - \underline{\epsilon}_t^1 \geq (1-\ell_d^1) f_t + \ell_d^1 g_t\} = 1$, $I_{hh}^{10}(m^1) = \{g_t - \bar{\epsilon}_t^1 \geq (1-\ell_d^1) f_t + \ell_d^1 g_t\} = 0$, $I_{lh}^{10}(m^1) = \{g_t - \underline{\epsilon}_t^1 \geq (1-\ell_d^1) f_t + \ell_d^1 g_t\} = 1$,

$I_{hl}^{10}(m^1) = \{g_t - \bar{\epsilon}_t^1 \geq (1 - \ell_d^1)f_t + \ell_d^1 g_t\} = 0$. Substituting into the above conditions yields the closure of R2.

The equilibrium is unique in the sense that it induces a unique distribution over outcomes among maximally informative equilibria. We defined signaling strategies in terms of arbitrarily chosen signals ($m^1 - m^4$) and have shown that the strategies assigned to arbitrary signals in R1-R3 are more informative than any other strategies that can survive in equilibrium. (Note that arbitrary signals $m^1 - m^4$ in one region need not be the same arbitrary signal in another region.) Although different specific signals can be used by different classes of types, this clearly does not induce a different distribution over outcomes. Therefore, since we have seen that the strategies assigned to the regions exist in the region assigned and not in any of the other regions, the equilibrium is unique in the sense described. ■

Condition 1 (C1) $\ell_t^1 < 1 - \frac{\epsilon_d^1}{g_d - f_d}$ and $\ell_t^2 < 1 - \frac{\epsilon_d^2}{g_d - f_d}$. In R1, $\ell_d^2 \leq 1 - \frac{\epsilon_t^2}{g_t - f_t}$; in R2, $\frac{\ell_d^1 \ell_d^2}{(1-\ell_d^1)(1-\ell_d^2)+\ell_d^1\ell_d^2} \leq 1 - \frac{\epsilon_t^1 + \epsilon_t^2}{g_t - f_t}$; and in R3, $\ell_d^1 \leq 1 - \frac{\epsilon_t^1}{g_t - f_t}$.

Proposition 3 Under A1 and C1, an equilibrium exists in which $(q_a \mid m) > q_a \; \forall m = a$.

Proof Let the arbitrary messages defined in Proposition 2, m^1 through m^4, be $10, 01, 11$ and 00 respectively. Then the Target's strategy in Proposition 2 implies $(q_a \mid m \neq a) = 0 \; \forall a \neq 00$, which implies $q_a = (q_a \mid a)\Pr(m = a) \; \forall a \neq 00$. Since $q_a > 0$ and $0 < \Pr(m = a) < 1 \; \forall a \neq 00$, and since $q_{00} = (1 - (q_{11} \mid 11))\Pr(m = 11) + (1 - (q_{10} \mid 10))\Pr(m = 10) + (1 - (q_{01} \mid 01))\Pr(m = 01) < 1 = (q_{00} \mid 00)$, we have $(q_a \mid m) > q_a \; \forall m = a$. ■

Corollary 1 Under A1, in Regions 1 and 3, an equilibrium exists in which for some m played with positive probability in equilibrium, $(p_a \mid m) = 1 \; \forall a \neq 11$.

Proposition 4 Under A1 and A2, if o such that $\ell_t^1 > 1 - \frac{\epsilon_d^1}{g_d - f_d} \vee \ell_t^2 > 1 - \frac{\epsilon_d^2}{g_d - f_d}$ then $n(h(o)) < 3 \; \forall h(o) \in H(o)$.

Proof $n(h(o)) < 4$ by Proposition 1. Proposition 2 demonstrated that there are only 3 possible messaging schemes that result in 3 signals being sent in equilibrium, namely, those assigned to R1-R3. But this implies that either equations (4)-(7) must hold or equations (8)-(11) must hold or equations (12)-(15) must hold. First, note that if $\ell_d^2 > 1 - \frac{\epsilon_t^2}{g_t - f_t}$, then $I_{y_t}^{01}(m^2) = 0\ \forall y_t$, which implies that equation (5) cannot hold. But if $\ell_d^2 \leq 1 - \frac{\epsilon_t^2}{g_t - f_t}$, (4) and (5) simplify to $\ell_t^2 \leq 1 - \frac{\epsilon_d^2}{g_d - f_d}$ and $\ell_t^1 \leq 1 - \frac{\epsilon_d^1}{g_d - f_d}$. By similar logic, conditions (12) and (13) imply the same. If $\frac{\ell_d^1 \ell_d^2}{(1-\ell_d^1)(1-\ell_d^2)+\ell_d^1 \ell_d^2} \leq 1 - \frac{\epsilon_t^1 + \epsilon_t^2}{g_t - f_t}$, then conditions (8) and (9) imply the same again. On the other hand, if $\frac{\ell_d^1 \ell_d^2}{(1-\ell_d^1)(1-\ell_d^2)+\ell_d^1 \ell_d^2} > 1 - \frac{\epsilon_t^1 + \epsilon_t^2}{g_t - f_t}$, the numerator of the RHS of conditions (10) and (11) is zero and these conditions cannot be satisfied. ■

Interpretation 1

Stage 1	**Stage 2**	**Stage 3**
Deterrer makes costless threats and assurances about conditions of war and peace	Target decides what concessions to make	Deterrer decides whether or not to go to war

Interpretation 2

Stage 1	**Stage 2**	**Stage 3**
Deterrer makes costless threats and assurances about agreement it will accept	Target decides what concessions to offer	Deterrer decides whether or not to accept the proposed offer

Figure 1

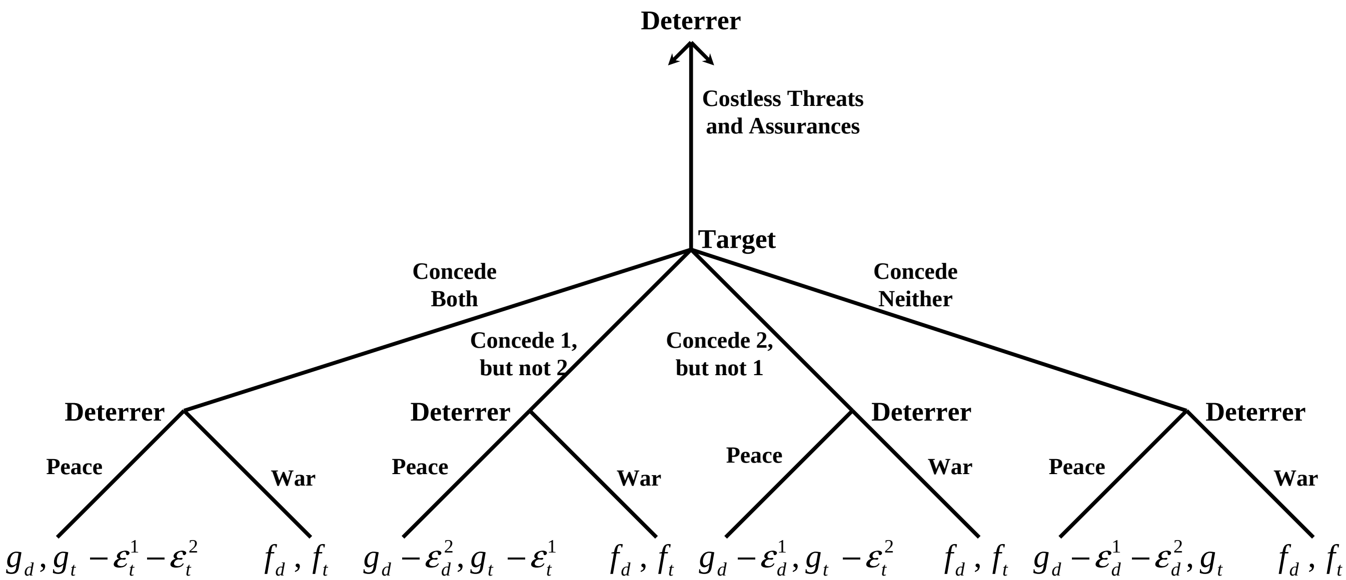
Deterrer
Costless Threats and Assurances
Target
Concede Both
Concede 1, but not 2
Concede 2, but not 1
Concede Neither
Deterrer
Deterrer
Deterrer
Deterrer
Peace
War
Peace
War
Peace
War
Peace
War
$g_d, g_t - \varepsilon_t^1 - \varepsilon_t^2$
f_d, f_t
$g_d - \varepsilon_d^2, g_t - \varepsilon_t^1$
f_d, f_t
$g_d - \varepsilon_d^1, g_t - \varepsilon_t^2$
f_d, f_t
$g_d - \varepsilon_d^1 - \varepsilon_d^2, g_t$
f_d, f_t

Figure 2

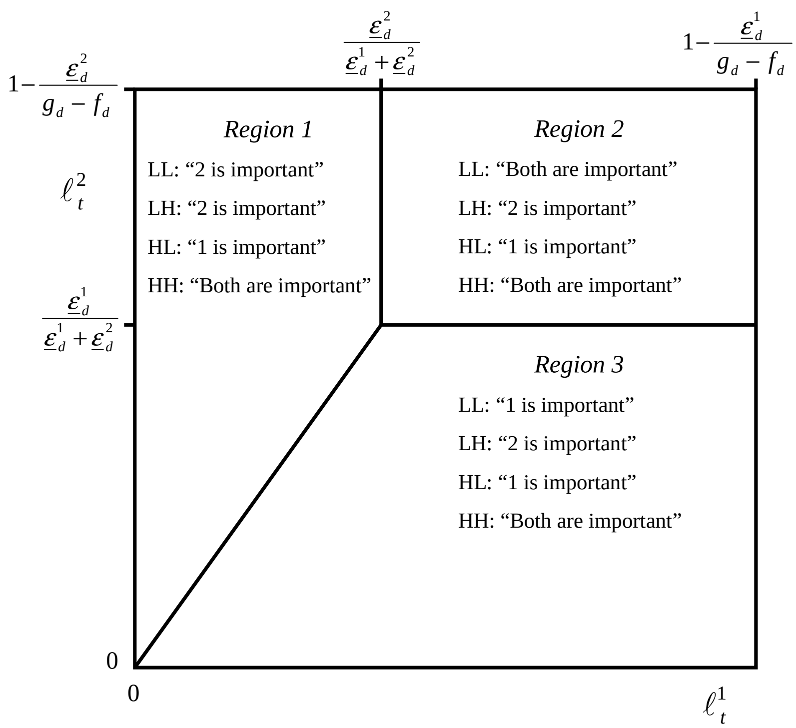

Figure 3: Deterrer Equilibrium Signaling Strategies

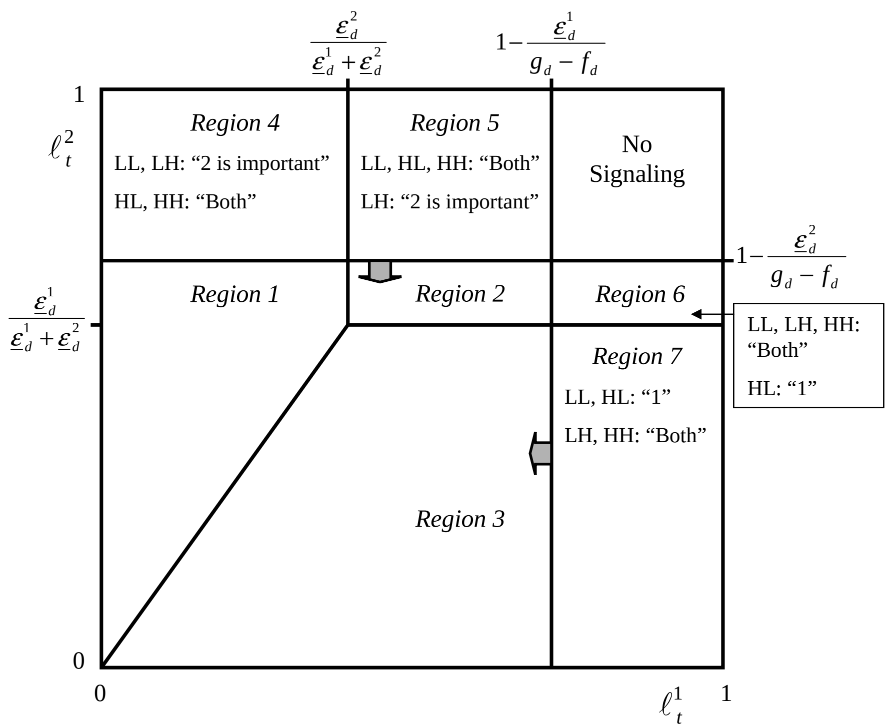

Figure 4: Plausible Signaling Equilibrium

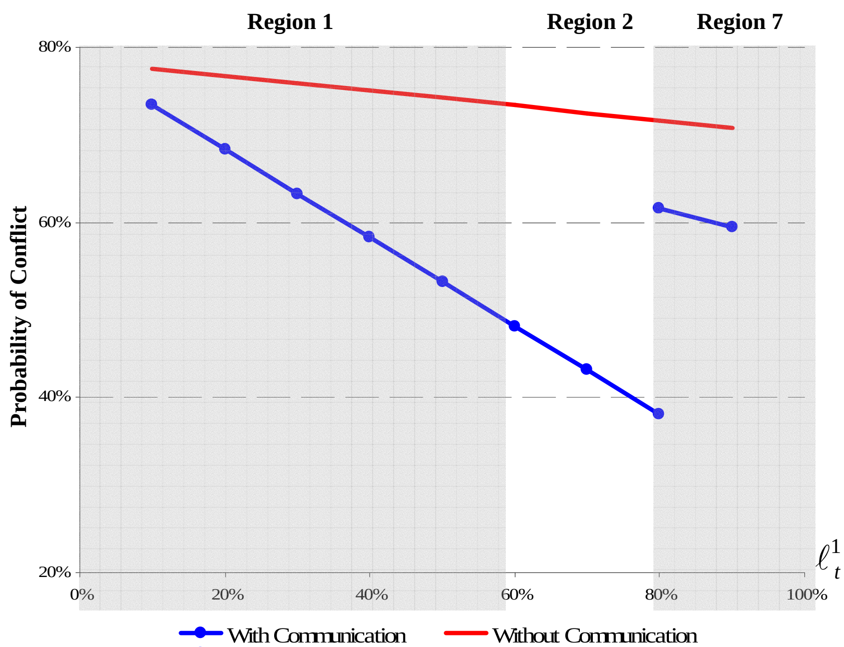

Figure 5: Effect of Likelihood of Target Resolve on the Likelihood of Conflict

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